Exercise 51

Find y' if $y = \ln(x^2 + y^2)$.

Solution

Differentiate both sides with respect to x.

$$\frac{d}{dx}(y) = \frac{d}{dx}\ln(x^2 + y^2)$$

$$\frac{dy}{dx} = \frac{1}{x^2 + y^2} \cdot \frac{d}{dx}(x^2 + y^2)$$

$$= \frac{1}{x^2 + y^2} \cdot \left(2x + 2y\frac{dy}{dx}\right)$$

$$= \frac{2x}{x^2 + y^2} + \frac{2y}{x^2 + y^2}\frac{dy}{dx}$$

Solve for dy/dx.

$$\frac{dy}{dx} - \frac{2y}{x^2 + y^2}\frac{dy}{dx} = \frac{2x}{x^2 + y^2}$$

$$\left(1 - \frac{2y}{x^2 + y^2}\right)\frac{dy}{dx} = \frac{2x}{x^2 + y^2}$$

$$\left[\frac{(x^2 + y^2) - 2y}{x^2 + y^2}\right]\frac{dy}{dx} = \frac{2x}{x^2 + y^2}$$

$$\left(\frac{x^2 + y^2 - 2y}{x^2 + y^2}\right)\frac{dy}{dx} = \frac{2x}{x^2 + y^2}$$

$$\frac{dy}{dx} = \frac{x^2 + y^2}{x^2 + y^2 - 2y}\frac{2x}{x^2 + y^2}$$

$$= \frac{2x}{x^2 + y^2 - 2y}$$